

Teaching Mathematics with Piaget's Theory

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Abstract

3 *Piaget's theory of cognitive development is a comprehensive theory about the nature and development of human intelligence first developed by Jean Piaget. It is primarily known as a developmental stage theory, but in fact, it deals with the nature of knowledge itself and how humans come gradually to acquire it, construct it, and use it. Moreover, Piaget claims the idea that cognitive development is at the centre of human organism and language is contingent on cognitive development. Below, there is first a short description of Piaget's views about the nature of intelligence and then a description of the stages through which it develops until maturity. This paper presented student's way of thinking in mathematics learning. Jean Piaget's theory brought here as the main concept applied on children thinking process in mathematics. Through observation, some fact emerges in every stage of development process. Those display on "Cant and But Can't" Table.*

Key words Mathematics learning, Piagetian, Piaget's Theory

Introduction

Usually in some courses there are several students who never seem to understand what is really going on. These students are neither lazy nor dumb. Some even work very hard in math and do quite well in their other courses. Yet there seems to be something about their work in mathematics which produces frustration instead of understanding. Then recall experiencing that kind of "learning" in high school geometry. Many students got fairly good grades in that course by memorizing without much understanding. As a high school sophomore they are not ready for deductive reasoning, proofs, axioms, etc. There were some essential mental skills which they had not yet developed.

As all college teachers know, the lack of certain specific mental skills is not restricted to high school students. Recent research can document that fact quite well, (Kohlberg and Gilligan, 1971; Lawson and Renner, 1974). To be more specific about mathematics consider the following examples of reasoning from freshmen.

Problem : Your instructor is 40. You are 18. In percent, how much younger than the instructor are you?

Student Solution 1: $18/40 = 0.45 \times 100 = 45\%$ younger.

Student Solution 2: $40/100 * 18/x = 720/100 = 7.2$ percent.

Problem: A rat going through a certain maze must take four decisions whether to turn left or right. One path he could take may be described, as RRLL, which means turn right, turn right, turn left, and finally turn left. List all possible paths the rat might take.

Solution: LLLL LLRR LRLR LLLR LRRL LRRR
RRLL RLRL RRRL RLLR RLLL RRRR

Theorem: If a series converges then the n -th term must tend to zero as n goes to infinity.

Student Application: Since $\lim 1/n = 0$, the harmonic series, $\sum 1/n$ converges.

What is striking about these examples is how accurately they are described in terms of Piaget's concrete operational and formal operational stages. Proportional reasoning, ability to conceive all possibilities, and use of basic propositional logic are three factors which distinguish the concrete and formal stages of development (Inhelder and Piaget, 1958, chapter 17).

Nature of intelligence: operative and figurative intelligence

Piaget believed that reality is a dynamic system of continuous change, and as such is defined in reference to the two conditions that define dynamic systems. Specifically, he argued that reality involves transformations and states. Transformations refer to all manners of changes that a thing or person can undergo. States refer to the conditions or the appearances in which things or persons can be found between transformations. For example, there might be changes in shape or form (for instance, liquids are reshaped as they are transferred from one vessel to another, humans change in their characteristics as they grow older), in size (e.g., a series of coins on a table might be placed close to each other or far apart) in placement or location in space and time (e.g., various objects or persons might be found at one place at one time and at a different place at another time). Thus, Piaget argued, that if human intelligence is to be adaptive, it must have functions to represent both the transformational and the static aspects of reality. He

proposed that operative intelligence is responsible for the representation and manipulation of the dynamic or transformational aspects of reality and that figurative intelligence is responsible for the representation of the static aspects of reality.

Operative intelligence is the active aspect of intelligence. It involves all actions, overt or covert, undertaken in order to follow, recover, or anticipate the transformations of the objects or persons of interest. Figurative intelligence is the more or less static aspect of intelligence, involving all means of representation used to retain in mind the states (i.e., successive forms, shapes, or locations) that intervene between transformations. That is, it involves perception, imitation, mental imagery, drawing, and language. Therefore, the figurative aspects of intelligence derive their meaning from the operative aspects of intelligence, because states cannot exist independently of the transformations that interconnect them. Piaget believed that the figurative or the representational aspects of intelligence are subservient to its operative and dynamic aspects, and therefore, that understanding essentially derives from the operative aspect of intelligence.

At any time, operative intelligence frames how the world is understood and it changes if understanding is not successful. Piaget believed that this process of understanding and change involves two basic functions: Assimilation and accommodation

Borrowing heavily from J. D. Herron (1975), L. Copes (1975) has given a list of tasks commonly expected in basic mathematics courses which he suspects most students not at the formal operations stage cannot do with understanding. Here is Copes' list.

Concrete-Operational Students

Can	But Can't
make routine measurements and observations.	measure "indirectly" quantities such as speed and acceleration, perhaps even area and volume.
answer acceptably the question, "Are there more squares or rectangles in the diagram"? if they realize that all squares rectangles.	respond correctly to the choice, "If all squares are rectangles, then: 1. All rectangles are squares; 2. Some rectangles are squares; 3. No rectangles are squares.
order a collection of sticks according to length.	decide who is tallest if told that Bill is taller than Tammy and shorter than Sheila.
count and perform elementary arithmetic operations.	systematize counting well enough to understand procedures permutations and combinations.
manipulate algebraic expressions including fractions.	given the equation $y = 3x^2$ or $y = 1/x$, decide what happens to y as x increases.
generalize simply from given data: all quadratic equations (in x) represent parabolas.	perform a "once-removed" generalization: since quadratic equations in x represent parabolas, so do quadratic equations in y .

From recent experience with freshmen, I would add that most concrete-operational students also

Can	But Cant
solve $x/3 = 7/5$ for x	find the shadow of a three foot child when his five foot mother has a seven foot shadow
change a number from base ten into base two using a memorized procedure	use the analogous process to write numbers in base three
write numbers in scientific notation	solve linear equations with coefficients written in scientific notation
apply memorized formulas to find the mean and standard deviation	decide whether the computed mean and standard deviation are at all reasonable for the data
find the intersection of two given finite sets	draw a Venn diagram to represent "some A are also B"
list all possible outcomes of flipping two coins	easily list the outcomes of three and certainly not for four coins
work through a flowchart with a given set of data points	discover that the flowchart just counts the positive data
write simple truth tables involving implication	gives a specific example of the denied antecedent fallacy
compute what percent 6 is of 8	find the percent change from 8 to 6.

Copes summarizes concrete operational students this way: "If I am at all correct, it follows that they are not able to follow a formal argument, much less to come up with a proof of their own. They cannot grasp the concept of a function, because the concept of variable is not clear. And, in terms of attitude toward our field of study, they certainly cannot appreciate playing mathematics, seen as a rule-oriented game...It should be clear by now that the students we are discussing are not stupid nor lazy. Perhaps they are not "reasoning" in our logical sense of the term, but we need to consider the possibility that this is due to gaps in the development of their mental structures rather than to inherent lack of growth potential."

The ability to use proportions when appropriate is an important and easily testable attribute of formal-operational reasoning. Especially in mathematics and science related courses it is essential for students to be able to use ratios. Yet how many students can find the shadow of the three foot child as suggested on the previous page? Data relative to this question were collected from several thousand freshmen through seniors in science courses throughout the country. The results (Thornton and Fuller, 1981) show a strong dependence on how the problem is stated. If the problem is given in verbal form, even with a diagram, only 60% will solve it correctly. If ratios are indicated by the format and numbers are used, close to 80% give the correct response. These data indicate definite gaps in the reasoning skills of many students.

Piaget's theory of intellectual development provides not only a very compelling model for the description of reasoning skills but also a possible prescription for student growth. In his article Piaget (1974) comments, "My first conclusion is that learning of structures seems to obey the same laws as the natural development of these structure. In other words, learning is subordinated to development and not vice-versa."

The doctor's orders seem to be: If the symptoms of the mathematics student are those of concrete operations we cannot treat him by working on the symptoms, that is by teaching

specific facts, concepts, methods. The cause must be treated. We must somehow change his level of development before satisfactory learning of certain mathematics content will be possible.

This prescription seems very difficult to take. It is much easier to restrict attention to mathematical concepts from a course syllabus than to be concerned about a student's entire intellectual capability. Discovering how a student reasons about a problem takes much more time than asking if he has a correct answer. Time spent on reasoning processes will restrict, sometimes severely, the amount of time available for mathematical content. An additional complication is that usually a student's mathematics class is just a small part of his academic load. Can one reasonably hope that what is done in one class will necessarily transfer to other courses and to his life outside the university?

Of course one cannot teach a mathematics course and expect to affect reasoning skills or anything else without including something to reason about. The ADAPT mathematics class was designed to contain mathematics which would encourage the students to consider their own reasoning and which would be useful in their other courses that semester. Content for the class was also chosen to be new and interesting to the freshman with a one to four years preparation in high school mathematics. The emphasis was not on formulas and correct answers but on the reasoning behind some formulas and how they could be applied. The following is a short description of the mathematical content of the original ADAPT math class during the academic year 1975-76. Content in succeeding years has been similar.

The class usually met for three fifty minute periods each week for both the fall and spring semesters. Several times during the first semester classes were held jointly with physics and these lasted two to three hours. The first part of the fall semester was to be a review of many mathematical ideas from high school. This material turned out to be new to many and thoroughly forgotten by most. Topics included functions with domain and range, inductive thinking, deductive thinking, similar triangles, percentage, number-numeral distinction, scientific notation, and a great deal of practice on linear equations, both graphing them and finding the equation from the graph. The work on scientific notation and linear equations covered a three week period when the same topics were considered in physics.

The middle of the semester began with work on logarithms. They were introduced using base 2, properties of logs were discovered and then extended to base ten. Both four place tables and pocket calculators were used for computation. A little over a week was spent on base two numeration, the binary sequence and its application to counting problems. Box puzzles were used to review arithmetical operations with fractions and decimals and to find the sum of an arithmetic series.

By this time the students had gathered lots of data in their anthropology course and had begun to see the need for statistical treatment. In mathematics they spent two weeks considering the notions of frequency distribution, mean, median, mode, and standard deviation. These ideas were then applied in their physics lab in predicting the pattern of falling pellets.

The last three weeks of the first semester were used to study unions, intersections, complements, deMorgan's laws, Venn diagrams and their use in organizing the displaying data for solving problems. Each student used a set of cardboard objects of various sizes, shapes, and colors to make their work in set theory more concrete. Some work was done in trying to draw Venn diagrams which represented logical connections expressed in English sentences.

Beginning logic was treated the first four weeks of the second semester. The operations from set theory were translated into logical connectives. Truth tables were done with three

independent propositions. Implication and the logical fallacies of the denied antecedent and the affirmed consequent were discussed. Almost a quarter of the students were interpreting p implies q as p is equivalent to q . This was expected in the light of O'Brien's findings (1973) on the use of implication by college students. Some work was done on syllogisms using Venn diagrams. All but a very few found syllogisms quite difficult.

The next four weeks treated significant figures and exponential functions. At this time in physics they were taking a lot of data related to exponential growth and decay. Mathematical problems were worked based on measured data to show that even though their calculator gives an answer with ten digits only the first so many are significant. Semi-log graph paper was used in graphing exponential functions. Material on logarithms and liner functions from the first semester was recalled to determine the exponential function from a straight line on semi-log paper. The experience on exponential functions was applied to compound interest problems.

The students spent three weeks working with flowcharts. Almost all were eventually comfortable following a given flowchart to make specific computations or decisions. Some students fairly easily constructed flowcharts for given processes or algorithms. Many students had trouble making flowcharts because the reasoning processes used to solve certain problems were not clear enough for them to write down in a flowchart format.

The last portion of the semester dealt with the idea of a mathematical model. Ideas and procedures from other courses as well as those in mathematics were interpreted as models. Models were constructed and applied to decision and prediction problems using the methods of least squares, linear programming, and critical path analysis.

It was very rewarding to see examples of how the content from this course was used in the students reasoning, especially when used outside the class. The most obvious examples of such transfer were in the statistical methods used in anthropology and in the applications of linear equations, exponential functions, graphing and computational skills used in physics. There were other less expected occasions. During the seminar when the students were given the results of the personality inventory they had taken, some discussion was not in terms of "Am I above or below average?", but rather, "Am I within a standard deviation of the mean?" In giving an explanation of the relationship between two attributes of human behavior in an anthropology journal, one student did it using a Venn diagram. In a history examination question one student responded by specifically mentioning the logical form of the argument. During an economics class a student was reporting on estimation of tax revenues based on percentage increases from previous years. Another student commented, "Hey. That makes it an exponential function."

How can I get my students to understand this concept? The instructor who views his students with Piaget's model of intellectual development can ask that question in a sharper form: How can I plan a basis of concrete activities and disequilibrating experiences so my students will develop adequate mental structures for this concept? In either form, my question is a hard one. You soon discover that what seems very concrete to you may not be concrete at all to your students. So the second question may be no easier to answer than the first. But with each way of asking the basic question, with each way of viewing how students can learn we gain an advantage for some students. Piaget's theory is effective for a great many students. Using it can help maximize the number of students who will say, "Hey, that looks like an exponential function."

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